1 Accuracy and precision

Generally, the difference between accuracy and precision is described in terms of firearms and target practice. A precise firearm is one that is built to strict tolerances, so that in the hands of an experienced marksman, can put five rounds into a one-inch grouping at 200 m. If the iron sights or scope is correctly adjusted, then that one-inch grouping can be put onto the bull's eye of the target; however, if the iron sights or scope are incorrectly adjusted, that same one-inch grouping would appear anywhere else other than the bull's eye. The Finnish Tikka T3x TAC A1 is one such firearm.

An imprecise firearm is one that is not built to strict tolerances, a common example being the Soviet AK-47. This firearm is designed to simply allow the user to put rounds downrange with high rates of fire with the intent of suppressing the adversary as opposed to necessarily being on target: the bolt holding the round in the chamber will still audibly rattle, meaning that when the trigger is pulled, there will necessarily be immediate vibrations that will cause the firearm to vibrate even before the round has left the barrel. An AK-47 can, never-the-less, be accurately sighted so that a skilled user can still put five rounds in a six-inch circle around the bull's eye at 100 m; however, no matter how skilled the marksman, it will not be possible to reduce that grouping significantly: the imprecise construction techniques result in a firearm that simply cannot reliably perform any better than the performance suggested.

A humorous story from Oktoberfest. At a Festhalle that will remain unnamed, one could test one's skill with an air rifle by trying to shot the noodle at the base of a flower to win that flower. After three carefully aimed shots, with each missing, this author became rather frustrated. The back stop was conveniently painted black, so one could not see the fall of one's shot and thus adjust; however, this author noted a yellow Post-It note that conveniently was attached to the back stop that was still in the direction of fire. Aiming at the center put the round almost a full inch to the left: the air rifle was not accurately sighted. This author then returned to aim approximately one inch to the right of each noodle on the next three flowers and took out the noodle in each of them. The air rifle was sufficiently precise to consistently hit a noodle at the prescribed distance.

This is however less optimal for engineering students, and thus, here we present a more concrete description of accuracy versus precision by describing the qualities of a sensor.

1.1 An alternate analogy

Suppose you have a sensor that periodically takes some sort of reading. This could motion sensors such as accelerometers, gravity sensors and gyroscopes, and rotational vector sensors; environmental sensors such as barometers, photometers and thermometers; and position sensors such as orientation sensors and magnetometers. [1]

In the following figures, a sensor is reading a physical and continuous signal (shown as a red line), but the sensor is only being read periodically (shown as blue dots). The sensor readings may or may not be close to the actual signal.

To begin, Figure 1 shows a relatively precise sensor that is also accurate, as each reading is close to the actual value. Such sensors tend to be more expensive than more imprecise sensors,

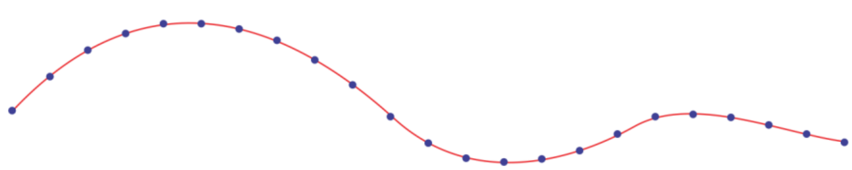

Figure 1. A relatively precise and accurate sensor.

Suppose, however, that the sensor in Figure 1 is not correctly calibrated, and it always gives a reading that is biased in one direction, such as shown in Figure 2. The values track the actual physical phenomenon being measured, but the measurement in this case is always too large.

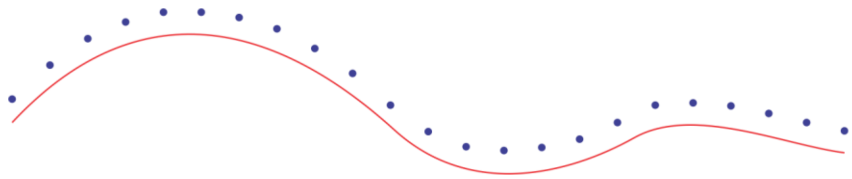

Figure 2. A relatively precise sensor that is never-the-less inaccurate.

A less expensive sensor may still be accurate, but may not be able to give as precise a reading, as is shown in Figure 3. One may be able to measure the precision of a sensor, but one may also be able to compare the relative precision of two different sensors. For now, we will refer to relative precision, but later there will be an optional topic discussing how we can measure precision.

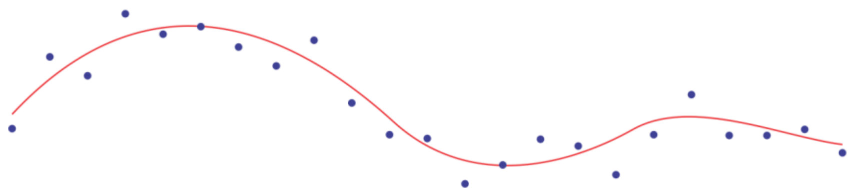

Figure 3. A less precise but still accurate sensor.

A less precise sensor may, never-the-less, also be inaccurate, as is shown in Figure 4. In this case, one may note that while some readings overestimate value of the true signal, almost 80% of the readings underestimate the exact value.

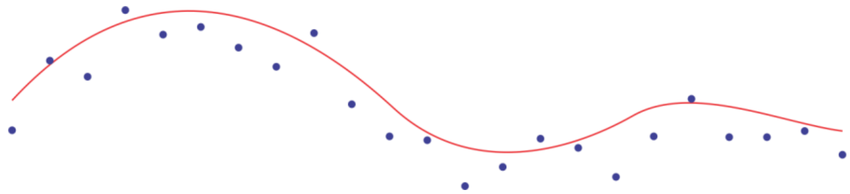

Figure 4. Readings form a relatively imprecise and inaccurate sensor.

In some cases, the output of an analog sensor (one where the reading is perhaps stored as a voltage), in other cases, that analog reading may be converted to a digital reading through an analog-to-digital converter (ADC). The output is usually a binary integer that is intended to be interpreted appropriately (perhaps as a fixed-precision floating-point number). Because each reading is rounded to the closest digital value, such digital readings are necessarily less precise than the original analog readings. For example, Figure 5 shows how the readings in Figure 1 may look if they are discretized.

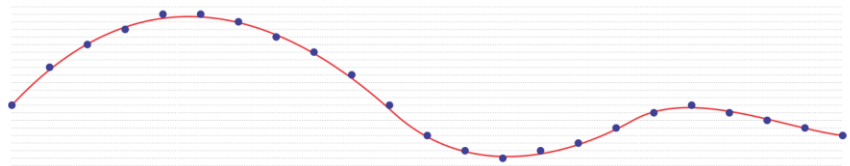

Figure 5. The readings in Figure 1 discretized only allowing values on the discrete collection of shaded lines.

In your courses on analog and digital circuits, you will learn how an analog signal can be converted into a digital signal. Of course, the greater the number of possible digital values, the more expensive the ADC will be and the more power it will use (although this is generally negligible).

Now, a sensor will not necessarily maintain its precision and accuracy throughout its lifetime. As components degrade over time, the precision may diminish, and as sensors are exposed to environmental conditions, they may become miscalibrated. In the following figures, we will describe such scenarios, but note that some are unrealistically compressed in time, and are given only as examples.

A sensor that experiences an environmental shock (lightning or a physical jarring) may become less accurate following that incident, as is shown in Figure 6.

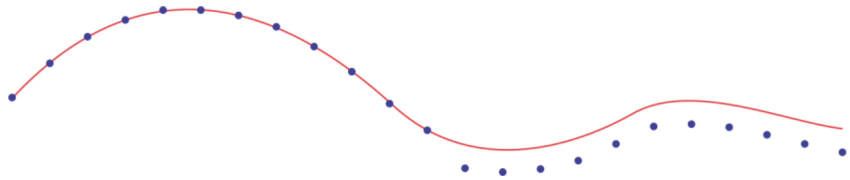

Figure 6. A drop in accuracy as a result of a sensor being exposed to a physical or electromagnetic shock or an extreme temperature.

The precision of a sensor may degrade over time as components within the sensor experience wear, as is shown in Figure 7.

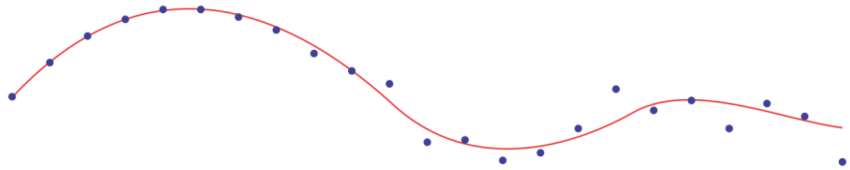

Figure 7. A sensor losing precision while still maintaining accuracy.

As a final example, a precise sensor may begin to give inaccurate readings due to drifts over time. As a first example, we see how the recorded values may drift over time, as shown in Figure 8. For example, a capacitor that should be discharged with each cycle may retain a small charge with each step, but over time, this increased charge stored in the capacitor may result in erroneous readings.

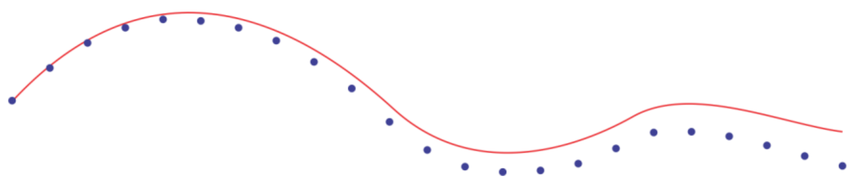

Figure 8. An example of how drift may cause a precise sensor to become inaccurate over time.

Alternatively, we may have a situation as is shown in Figure 9 where loss of synchronization may result in an increase in accuracy. For example, if a sensor is linked to a clock that uses a quartz crystal to track time, if the time on the clock is not periodically corrected, it will begin to drift, and thus, if the sensor is indicating that the reading was taken at 15:20:49, but the actual time of the reading is 15:21:03, the reading may be precise, but not accurate.

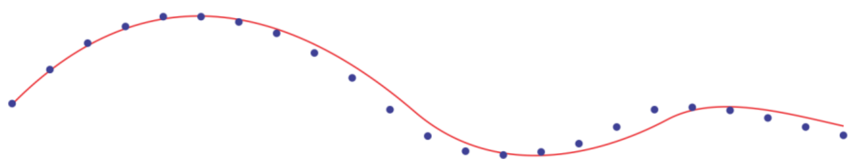

Figure 9. An example of how loss of synchronization may cause a precise sensor to become inaccurate over time.

1.2 Determining precision and accuracy (optional)

Suppose you have a sensor and you would like to know how precise and how accurate it is. This could be done in a laboratory setting by providing an input the exact value of which is known. We will then sample the sensor by periodically measuring both the phenomenon and the sensor reading and contrasting these:

1. Let x_1, x_2, x_3, … be the exact value being measured at the times the sensor is reading the phenomenon, and
2. let s_1, s_2, s_3, … be the corresponding sensor readings.

Recall that *exact* = *approx.* + *error*, and thus, *error* = *exact* – *approx.*, so let

$$e_1 \leftarrow x_1 - s_1,\ e_2 \leftarrow x_2 - s_2,\ e_3 \leftarrow x_3 - s_3,\ \ldots$$

be the errors of the sensor readings. Recall that $e_k \leftarrow x_k - s_k$ indicates that e_k is being assigned the difference on the right-hand side. In your course in statistics you will learn more about the average value and the standard deviation; however, for now, we will simply define the average value and standard deviation of such samples.

1.2.1 The average value (optional)

The average value e of n such errors will be defined to be

$$e = \frac{1}{n}\sum_{k=1}^{n} e_k,$$

This is a measure of a trend in the size of the errors, and if the average value of the errors is close to zero, then the sensor will be considered accurate, while a large average value will indicate the sensor is less accurate. The first is a measurement of which direction the errors lie, so if the mean is negative, the sensor readings generally giving higher readings than they should, while if the mean is positive, the sensor readings are too low.

To determine if a specific non-zero average value indicates that a sensor is definitely inaccurate (perhaps incorrectly calibrated or perhaps ill maintained), we must also look at the standard deviation of the errors.

1.2.2 The standard deviation (optional)

The "standard deviation" s of n such errors to be

$$s = \sqrt{\frac{1}{n-1}\sum_{k=1}^{n} \left(e_k - e\right)^2}.$$

Note: the n – 1 comes from the fact that the average value is only an approximation of the actual inaccuracy of the sensor (we have only sampled the sensor n times), and not the true inaccuracy of the sensor.

The standard deviation is a measure of how *spread out* the errors are relative to the average value. If all the errors are very close to the average value, then the standard deviation will be very small: the sign of a precise device. If the standard deviation is larger, this will indicate a more imprecise device.

1.2.3 Testing accuracy (optional)

To determine if a sensor is actually inaccurate, versus simply imprecise, we need to do more than just look at the errors. Suppose we have taken $n \geq 30$ readings from the sensor. To be 95% sure that the sensor is indeed inaccurate, we can perform the following calculation:

$$z \leftarrow \frac{e}{s/\sqrt{n}}.$$

If $|z| > 1.96$, then the sensor is inaccurate. If we are just beginning to work with the sensor in the laboratory, this may indicate a calibration issue. In the laboratory, however, we may subject the sensor to either

1. physical or electromagnetic shocks or extreme changes in temperature, or
2. long-term exposure to physical or electromagnetic forces, or changes or extremes in temperature.

Following this, we may then perform such a test to ensure the sensor is still sufficiently precise and accurate.

References

[1] Sensors Overview https://developer.android.com/guide/topics/sensors/sensors_overview , retrieved 2023-01-26.